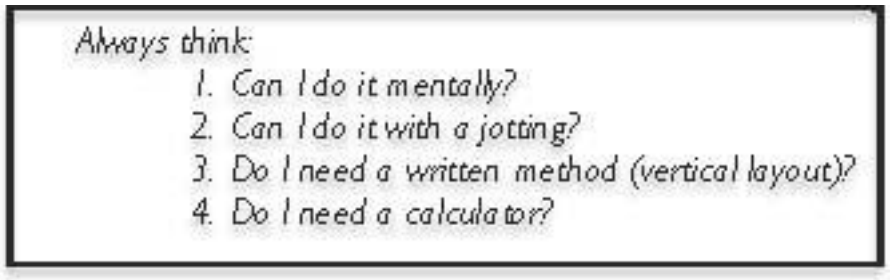
Calculation Policy

2014 National Curriculum for Mathematics

A guide for teachers and parents: developing children's number sense in mental and written calculation



Rationale

The following policy has been developed after consultation with the teachers of Goodmayes Primary School, the Redbridge Maths Team and Maths Specialist Teaching Instructors (MaST).

It is the intention of this document that the progression in strategies taught is understood alongside the central aim of developing 'number sense': nurturing children's ability to be flexible with using number. It is important then that the policy is not seen as a series of methods to be 'got through', but rather, guidance for strategies that may be used in service of this wider aim. As such, not every stepped approach will be necessary for every child. However, if a child is finding a particular strategy challenging, this may equally signify a conceptual misconception to be addressed.

In effective maths learning, procedural fluency in a method must be accompanied by a conceptual understanding of its meaning. As such, standard formal written methods such as 'short division' or 'column method' can be problematic if not fully understood. It is for this reason that written methods must be based on solid foundations of mental strategies, which in turn are supported through the use of manipulatives, images, models and representations. An established aim of the new mathematics curriculum is that children should learn to reason, explain and justify the types of maths they use by explaining not just what they are doing, but why and how the particular strategy works. Talk and reasoning should therefore be considered as a fundamental part of learning to calculate.

The policy of Goodmayes Primary is that manipulatives, images, models or representations should be available to children throughout primary school for as long as they are needed. Please see the appendix of this document for some more detail around which of these could be used to support the different stages of progression in calculating. The students of Goodmayes Primary School are also developing a suite of videos to demonstrate the strategies outlined in the stages below.

ADDITION GUIDELINES

Stage One

Prerequisite skills (based on the practical)
Counting numbers to 20

(using familiar / practical resources)

Place numbers to 20 in order

Bonds up to 10 and to make 10

1 more than a number

Number bonds to 20

Addition as combining groups

1, 2, 3, 4, 5, 6, 7

Addition as counting on

Doubling numbers within 20

Stage Two

Prerequisite skills (based on the practical)

Relate number bonds to 10 to add multiples of 10 up to a total of 100 e.g. if 3 + 4 is 7 then 30 + 40 is 70

Use familiar objects to recognise the place value of 2 digit numbers. (straws/ bead strings/ pegs)

Recognise and explain 24 is '2 tens and 4 ones'

Progressing to: **PARTITIONING AND RECOMBINING**
Partition into tens and ones and recombine Pre J10
(before jumping in 10s)

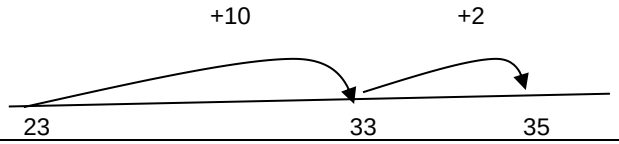
12 + 23 = 10 + 2 + 20 + 3
= 30 + 5
= 35

Model this on a bead bar and practise on 100-beadstrings, showing the 'collection' of 10s and then the ones. i.e. "2 tens and 1 ten makes 3 tens, which is 30. Then 3 and 2 makes 5 ones. Altogether we can see 3 tens and 5 ones, which is 35." Check by counting in tens and ones along the bead bar. Model and practise with place value arrow cards, numicon, bead strings or Dienes, using known facts and place value to calculate each step.

Count on in tens and ones
J10 (jumping in 10s)

23 + 12 = 23 + 10 + 2
= 33 + 2
= 35

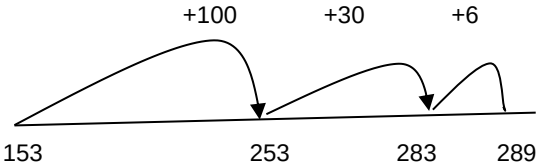
Model this on a number line starting at 23 and jumping 10 (J10) to make 33 and then add 2 in one jump.



Stage Three

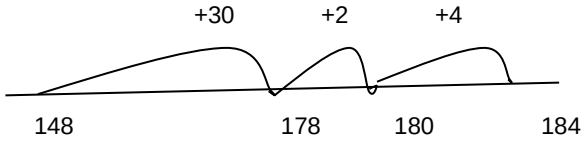
Partition into hundreds, tens and ones
• Partition one number and recombine.
• Count on by partitioning the second number only e.g.
136 + 153 = 153 +100+ 30 + 6
= 253 + 30 + 6
= 283+6
= 289

As modelled below as necessary



Children need to be secure adding multiples of 10 and 100 to any three-digit number including those that are not multiples of 10.

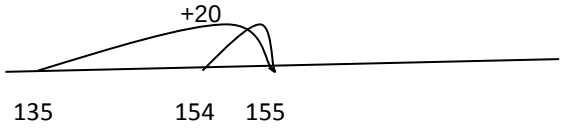
148 + 36 = 184



First J10 then T10

Add a near multiple of 10 to a two-digit number (Overjumping - OJ)

Secure mental methods by using a number line to model the method. Continue as in Stage 2 but with appropriate numbers E.g. 135 + 19 is the same as 135 + 20 – 1.



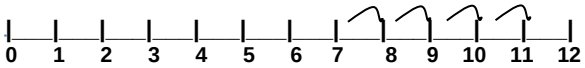
ADDITION GUIDELINES

Stage One

The Number Line

Children use a numbered line to count on in ones. Children use number lines and practical resources to support calculation and teachers *model* the use of the number line.

e.g. 7+ 4:



Number line Teaching Points:
Always work with numbers reading from left to right (smallest to largest), whatever the operation of the calculation. Numbers ('landmarks') are written below the line. Size of the 'jumps' are written above the 'jumps'.

The Empty Number Line:

Children use an empty number line to count in ones, twos and fives.
Children use an empty number line to add a one digit number to another number.

8+3=

+ = signs and missing numbers

Children need to understand the concept of equality before using the '=' sign. **Calculations should be written either side of the equality sign so that the sign is not just interpreted as 'the answer'.**

2 = 1+ 1
2 + 3 = 4 + 1
3 = 3
2 + 2 + 2 = 4 + 2

Missing numbers need to be placed in all possible places.

3 + 4 = = 3 + 4
3 + = 7 7 = + 4
+ 4 = 7 7 = 3 +
+ ∇ = 7 7 = + ∇

Stage Two

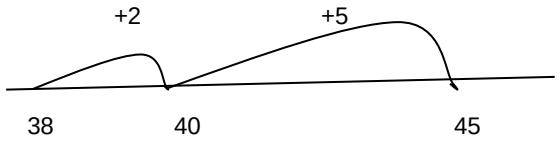
The Empty Number Line:
T10 (Targeting the 10, partitioning and bridging through 10)

The steps in addition often bridge through a multiple of 10

e.g.

Children should be able to partition the 7 to relate adding the 2 first to target the 10 and then the 5.

38 + 7 = 45



O10 may be used as a strategy in mental methods.

+ / = signs and missing numbers
Continue using a range of equations as in Stage 1 but with appropriate, larger numbers.
Extend to
14 + 5 = 10 +
and
32 + + = 100 35 = 1 + + 5

Stage Three

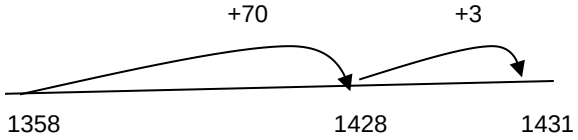
Once a child is able to add 3 digit numbers on a number line securely move on to vertical expansion.

+ / = signs and missing numbers Continue using a range of equations as in Stage 1 and 2 but with appropriate, larger numbers

ADDITION GUIDELINES

Stage Four

Partition into hundreds, tens and ones and recombine
Either partition both numbers and recombine or partition the second number only e.g.
1358 +73 = 1358 + 70 + 3
= 1428 + 3
= 1431



Horizontal Expansion
Children should add from right to left adding the least significant digit first. (ones)

367 + 185 = 552

```
  367
+185
   12 (7+5)
 140 (60+80)
 400 ( 300+100)
 552
```

Moving on to

```
  367
+185
   12
 140
 400
 552
```

(Without brackets)

Read' the answer from left to right, using knowledge of place value and referring to the value of each digit i.e "five hundred and fifty two"
NOT adding up columns for the final answer

It is crucial to know or be able to derive key number facts TU + TU mentally or with jottings before progressing to Stage Five.

+ / = signs and missing numbers Continue using a range of equations as in Stage 1 and 2 but with appropriate numbers.

Stage Five

Horizontal Expansion
Adding the least significant digits first

```
  2247
+ 1376     (without use of brackets)
    13
   110
   500
  3000
  3623
```

Moving on to a compact method

```
  2247
+ 1376
  3623
   1 1
```

Working from right to left:
"7 + 6 is 13. Partition the 13 into 10 and 3,'carry' the ten into the tens column, writing it as 1 to represent one ten."
n.b. the '1' can be written at the top or bottom of the calculation.

It is NOT "carry the 1"

Revert to expanded methods if the children experience any difficulty.

Consolidation and practice of the previous key facts. N.B. Please refer to the end of year expectation for the size and range of numbers to be using e.g. ThHTU, decimals, etc.

Stage Six

Extend to numbers with six digits

543,587 + 3,675 =

```
  543587
+   3675
  547262
   1 1 1
```

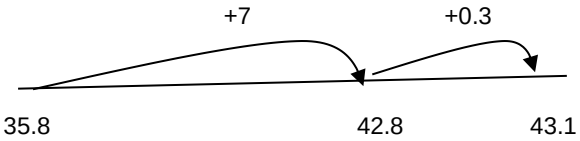
or decimals with up to 3 places
13.86 + 9.481 = 23.341

```
 13.86
+ 9.481
 23.341
 1 1  1
```

Revert to expanded methods if the children experience any difficulty.

Partition into hundreds, tens, ones and decimal fractions and recombine

Either partition both numbers and recombine or partition the second number only e.g.
35.8 + 7.3 = 35.8 + 7 + 0.3
= 42.8 + 0.3
= 43.1



Extend to up to two places of decimals (same number of decimals places) and adding several numbers (with different numbers of digits).

N.B. Please refer to the end of year expectation for the size and range of numbers to be using e.g. ThHTU, decimals, etc.

End of Year Objectives for Addition

Yr1 – recall and jottings for U+U, T+U, T+T, TU+U (within 20 including 0)

Yr2 – TU+U, T+TU, TU+TU, U+U+U

Yr3 – mental methods for HTU + U, HTU+T, HTU+H; written methods for HTU+TU, HTU+HTU

Yr4 – written methods as above and ThHTU+ThHTU, U.t+U.t, £U.th+£U.th

Yr5 – written method for addition of numbers with more than four digits; 2 or more integers, decimals with 2dp e.g. 29.78 + 54.34

Yr6 – written method for addition of numbers with more than six digits; 2 or more integers, decimals with 2dp

Differentiation Steps for each Stage:

- Not crossing tens
- Crossing Tens
- Crossing Hundreds Only
- Crossing Tens and Hundreds

In addition:

- The number line must be modelled as an image to support calculation from Reception to Year 6.
- Jottings must be modelled as a clear image/strategy for mental calculation.
- If the calculation can be carried out mentally then do not give it to practise vertical calculation, e.g.TU + TU should not be calculated vertically.

Always present calculations horizontally in order to consider mental calculations first.

SUBTRACTION GUIDELINES

Stage One

Prerequisite skills (based on the practical)
Number bonds to 10!

Counting back from 20

Find one less than a given number

Subtract using quantities and objects 2 single digit numbers

Count back to subtract single digit numbers

There are two concepts linked to subtraction:
Subtract - where it is natural to count back to 'take away'
Find the difference – where the understanding of the vocabulary leads to using addition to count on [complementary addition].

Understand subtraction as 'take away'

Stage Two

There are two concepts linked to subtraction:
Subtract - where it is natural to count back to 'take away'
Find the difference – where the understanding of the vocabulary leads to using addition to count on [complementary addition].

Use known number facts and place value to subtract

Using knowledge of number bonds to subtract mentally from multiples of 10s e.g. 30 – 4

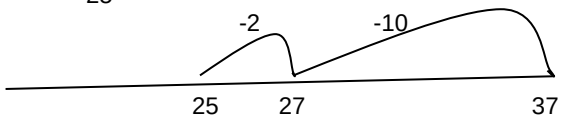
Using knowledge of number bonds to subtract mentally multiples of 10 from multiples of 10 e.g. if 7 – 4 = 3 then 70 – 40 = 30

Using knowledge of number bonds to subtract mentally e.g. if 8 – 3 = 5 then 28 – 3 = 25

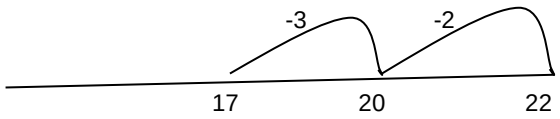
Use of J10 using multiples of 10
Example: 80 – 30

Use of J10 on an empty number line
Start by putting largest number on the right hand side and jumping backwards towards the left hand side of the number line.

37 – 12 = 37 – 10 – 2
= 27 – 2
= 25

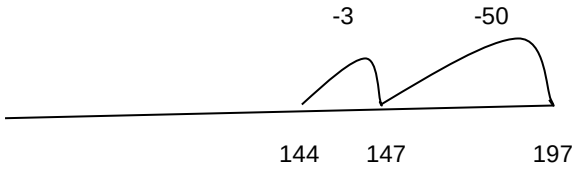


Use of T10 for TU-U on an empty number line
22 – 5 = 22 – 2
= 20 – 3



Stage Three

Use known number facts and place value to subtract
Continue as in Stage 2 but with appropriate numbers e.g.
197 – 53 = 144

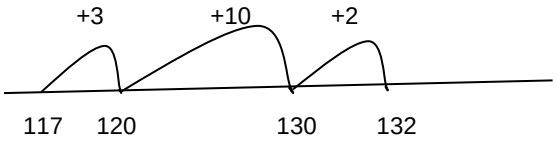


Secure knowledge in use of J10 and T10 to count back
TU-TU, HTU-TU, HTU-HTU

***By the end of this stage children should know complements to 100. They can then use this knowledge to calculate* HTU-TU, HTU-HTU.**

Subtract *mentally* a 'near multiple of 10' to or from a two-digit number
Continue as in Stage 2 but with appropriate numbers e.g. 78 – 49 is the same as 78 – 50 + 1

Use counting on to find the difference
132 – 117 = 15



- = signs and missing numbers(inverse)
Continue using a range of equations as in Stage 1 and 2 but with appropriate numbers.

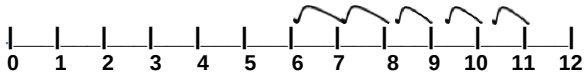
SUBTRACTION GUIDELINES

Stage One

Use practical and informal written methods to support the subtraction of a one-digit number from a one digit or two-digit number and a multiple of 10 from a two-digit number. I have 11 toy cars.

I lost 5 of them. How many are left?

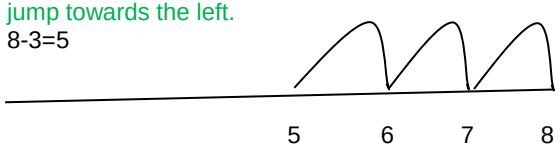
Start with bead strings / bars and move onto number lines below.
11-5=6



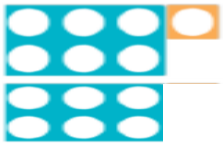
Use the vocabulary related to subtraction and symbols to describe and record subtraction number sentences.

The Empty Number Line:

Children use an empty number line to subtract by counting back in ones. Children to start from the right hand side and jump towards the left.
8-3=5



Use practical resources to find the difference between two small numbers e.g. 6 and 7



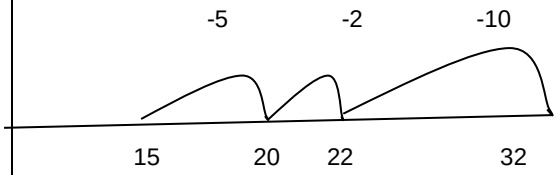
Count on from s largest number to find the difference where numbers are close in value. (e.g.9-7)

- = signs and missing numbers(inverse)

7 - 3 = ! ! = 7 - 3
7 - ! = 4 4 = ! – 3
! - 3 = 4 4 = 7 - !
! - = 4 4 = ! -

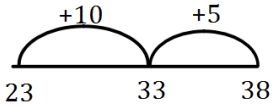
Stage Two

Use of T10 where necessary
32 – 17=



Subtraction for finding the difference using counting on

e.g. 38 – 23



- = signs and missing numbers(inverse)
Continue using a range of equations as in Stage 1 but with appropriate numbers.
Extend to 14 + 5 = 20 - (inverse)

Stage Three

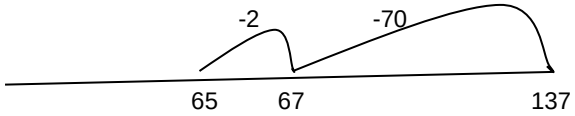
SUBTRACTION GUIDELINES

(-= signs and missing numbers: Continue using a range of equations as in Stage 1 and 2 but with appropriate numbers.)

Stage Four

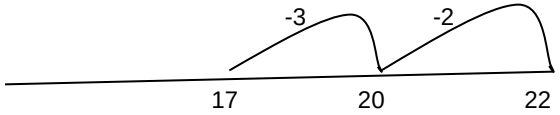
Use of J10 on an empty number line

137 – 72 =137 – 70 – 2
= 137 -70
= 67-2



Use of T10 for TU-U on an empty number line

322 –140 = 322-100
=222-20
= 202- 20
= 182



SUBTRACTION BY EXPANDED DECOMPOSITION

(With higher attainers secure in number facts and use of the number line).

Subtracting with no repartitioning needed:

345 – 123

3 0 0 4 0 5

- 1 0 0 2 0 3

2 0 0 2 0 2

Counting on

Use of number facts to count up to find the difference (T10, T100). 754 – 568 = 186

Find a small difference by counting up (relating to inverse)

e.g. 5003 – 4996 = 7

This can be modelled on an empty number line (see

Stage Five

SUBTRACTION BY EXPANDED DECOMPOSITION

Subtracting with no repartitioning needed:

345 – 123

3 0 0 4 0 5

- 1 0 0 2 0 3

2 0 0 2 0 2

Partitioning each number and working from right to left, subtracting the bottom number form the top. Express each part as its value represented, i.e. “40 – 20”.

Moving onto subtracting with repartition of tens only:

252 – 114

2 0 0 + 5 0 + 2
-(1 0 0 + 1 0 + 4)
?

4 0 1 2
2 0 0 + ~~5 0~~ + ~~2~~
-(1 0 0 + 1 0 + 4)

1 0 0 + 3 0 + 8

Again, partitioning each number and working from right to left, subtracting the bottom number from the top. Where the subtraction is not possible i.e. 2 – 4 can’t be done, the next value is “REPARTITIONED”. So, “repartition 50 + 2 into 40 + 12”. It is important to cross out the whole number and replace completely. Do NOT put a ‘one in the air’! (It is not a 1, it is a 10.) Then repeat the subtraction process, this time “12 – 4 = 8” and “40 – 10 =30”
This should be done using the place value counters alongside written methods.

Stage Six

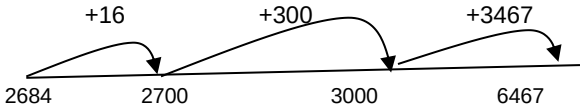
Progress to six digit numbers
Subtraction by Standard Decomposition

4346 – 128=

3 1
4 3 ~~4~~ 6
- 1 2 8
4 2 1 8

Counting on

6467 – 2684 = 3783



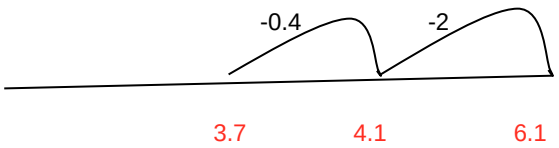
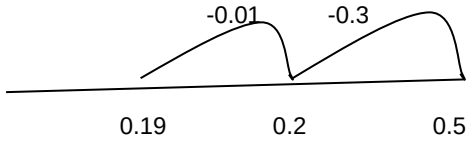
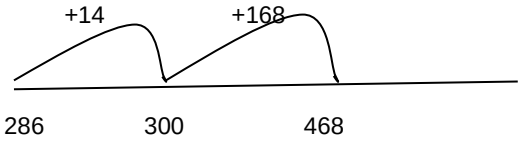
Reduce the number of steps to make the calculation more efficient.
Extend to 2 places of decimals

It is still vital that the correct language of place value is used. The tens are REPARTITIONED (not “’borrow’ a 1” and it is not “3 takeaway 1” but “300 takeaway/subtract/ minus 100”).

N.B. Please refer to the end of year expectation for the size and range of numbers to be using e.g. ThHTU, decimals, etc.

SUBTRACTION GUIDELINES

(-= signs and missing numbers: Continue using a range of equations as in Stage 1 and 2 but with appropriate numbers.)

Stage Four	Stage Five	Stage Six
complementary addition). Children should be encouraged to use known number facts to reduce the number of steps. **Use known number facts and place value to subtract** 6.1 – 2.4 = 3.7  **Use known number facts and place value to subtract** 0.5 – 0.31 = 0.19  ***N.B. Please refer to the end of year expectation for the size and range of numbers to be using e.g. ThHTU, decimals, etc.***	Counting on Use of number facts to count up to find the difference (T10, T100) this is used in the context of inverse. 754-286= 468  ***N.B. Please refer to the end of year expectation for the size and range of numbers to be using e.g. ThHTU, decimals, etc.***	

End of Year Objectives for Subtraction

Year 1 – mentally subtract U-U, TU-U, TU-TU (up to 20 e.g. 15 – 12)
Year 2 -mentally TU-U, TU-multiple of 10, mentally with informal jottings TU-TU
Year 3 – subtract mentally, HTU – U, HTU – T, HTU – H, TU-U, TU-TU. Formal written methods for TU-TU, HTU-TU, HTU-HTU
Year 4 – as above and efficient written methods for ThHTU-ThHTU, ThHTU-HTU, U.t – U.t, £U.th-£U.th
Year 5 – Efficient written methods for subtraction of 2 integers with more than 4 digits e.g. 45230 -12432 and decimals with up to 2dp e.g. 54.34-29.78
Year 6 – as above with numbers up to six digits

Please note: There are two concepts linked to subtraction: Subtract -where it is natural to count back to 'take away' Find the difference – where the understanding of the vocabulary leads to using addition to count on [complementary addition].

- Children should not move on to a written method if they are not completely confident with using a number line.
- Children will need to have had experience of different types of jumping on a number line e.g.T10 (target the ten), J10 (jump in 10s) and know how to partition numbers in different ways.
- These methods can also be easily applied, at different levels, to finding differences in values of money, measures and time.

Always present calculations horizontally in order to consider mental calculations first.

MULTIPLICATION

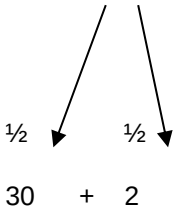
MENTAL STRATEGIES

Strategies to calculate the facts not yet recalled ARE essential:

× 2 double ÷ 2 halve
× 4 double-double ÷ 4 half and half again
× 8 double-double-double ÷ 8 half, half and half again

Model jottings for halving and doubling and use known facts and place value

½ of 6 4

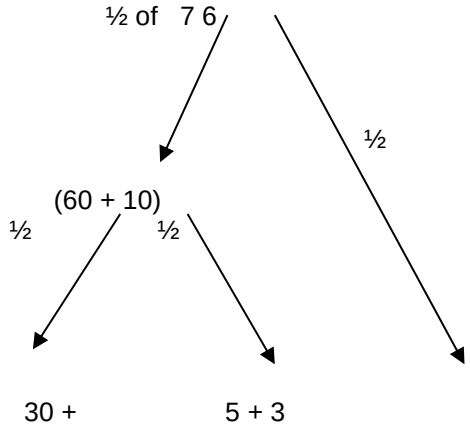


"Half of 6 tens or half of 60 is 3 tens or 30"

"Half of 4 is 2."

Where the number of tens (or hundreds) is odd and the fact unknown, use known facts to derive the new fact:

Eg



× 5 ½ of × 10

× 50 ½ of × 100

× 25 ¼ of × 100 (or ½ and ½ again of × 100

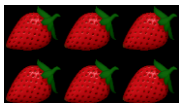
× 12 × 10 plus × 2 (double)

× 15 × 10 plus ½ of × 10

MULTIPLICATION GUIDELINES

Stage One

Prerequisite skills (based on the practical)
Multiplication is related to known facts including doubling and counting groups of the same size.



3 + 3
E.g. use of dominoes and dice.

Counting using a variety of practical resources

Numicon and bead strings

Counting in 2s e.g. counting socks, shoes, animal's legs…

Counting in 5s e.g. counting fingers, fingers in gloves, toes…

Counting in 10s e.g. fingers, toes…

Pictures / marks

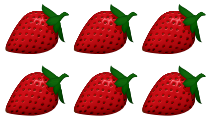
There are 2 socks in a pair
How many socks are there in 3 pairs?

Stage Two

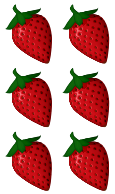
x = signs and missing numbers

7 x 2 = □	□ = 2 x 7
7 x □ = 14	14 = □ x 7
□ x 2 = 14	14 = 2 x □
□ x ∇ = 14	14 = □ x ∇

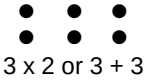
Arrays



Looking at rows
3 + 3
2 groups of 3



Looking at rows
2 + 2 + 2
3 groups of 2

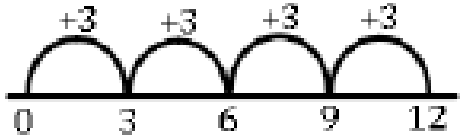


3 x 2 or 3 + 3

2 +2 + 2 or 2 x 3

Repeated addition

Repeated addition on an empty number line:



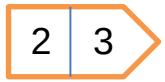
If the calculation is 3 x 4 for example, children should understand that this means 3 + 3 + 3 + 3. Children should also understand the commutative law and be able to use 4 x 3.

Stage Three

x = signs and missing numbers
Continue using a range of equations as in Stage 2 but with appropriate numbers.

Partitioning

Children need to be secure with partitioning numbers into 10s and 1s and partitioning in different ways: 6 = 5 + 1 so e.g. Double 6 is the same as double five add double one.



X 3 = 69

X 3 = 60

3 x 3 =9

Grid Method:

Partition 23 x 3 =

X	20	3
3	60	9

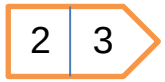
Moving on to:

x	10	6
10	100	60
7	70	42

100 + 60 + 70 + 42 = 272

At the end of Stage 3 the children should use the above strategies, as well as doubles of multiples of 5 and knowing the 2, 3, 4, 5, 6, 8 and 10 times tables from memory.

MULTIPLICATION GUIDELINES

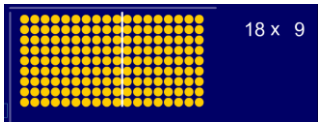
Stage One	Stage Two	Stage Three
At the end of Stage 1 the children should use the above strategies, as well as doubles and halves up to 20 and counting in 2, 5 and 10s. Children should be able to count in ten from any given number.	**Partitioning** Children need to be secure with partitioning numbers into 10s and 1s and partitioning in different ways: 6 = 5 + 1 so e.g. Double 6 is the same as double five add double one.  X 3 = 69  X 3 = 60 3 x 3 =9 ***At the end of Stage 2 the children should use the above strategies, as well as doubles of multiples of 5 and knowing the 2, 5, and 10 times tables from memory.***	

MULTIPLICATION GUIDELINES

Stage Four

x = signs and missing numbers
Continue using a range of equations as in Stage 3 but with appropriate numbers

Partition
Continue to use arrays:



18 x 9 = 162

18 x 9 = (10 x 9) + (8 x 9) = 162

Use Multiplication array ITP to model partitioning into tens and ones, using the familiar visual pattern of 5s.

Grid Method

Use the grid method of multiplication (as below)

36 x 27 =

x	30	6
20	600	120
7	210	42

600 + 120 + 210 + 42 = 972

At the end of Stage 4 the children should know their 12 x 12 times tables.

Stage Five

Partition
47 x 6 = 282

47 x 6 = (40 x 6) + (7 x 6) = 282

OR

Use the grid method of multiplication (as below)

Grid method
72 x 38 is approximately 70 x 40 = 2800

Remember, always present calculations horizontally in order to consider mental calculations first.

Again, if the calculation should be possible mentally then do not give it to practise vertical calculation, e.g. 23 x 15 should not be calculated vertically. Consider use of numbers carefully. Avoid numbers which involve x 2, x 4, x 5, x 8 which can be solved mentally using known facts.
382 x 23 =

x	300	80	2
20	6000	1600	0
3	900	240	6

6000 + 1600 + 900 + 240 + 240 + 40 + 6 = 8986

6000 + 2500 + 480 + 46 = 8000+ 980 +46

It is important to write the calculation in the grid for both the pupil and teacher to be able to identify errors made in multiplication facts or in the calculating the process. Where possible, use mental calculation strategies to calculate the total e.g. looking for known facts or adding the largest number first.

Stage six

Use the grid method of multiplication (as before))
Grid method
372 x 24 is approximately 400 x 20 = 8000

Extend to decimals with up to two decimal places.

Short Multiplication
The recording is reduced further, with carry digits recorded below the line.

38
x 7
266
5

Children who are already secure with multiplication for TU × U and TU × TU should have little difficulty in using the same method for HTU × TU or applying decimals.

Long multiplication

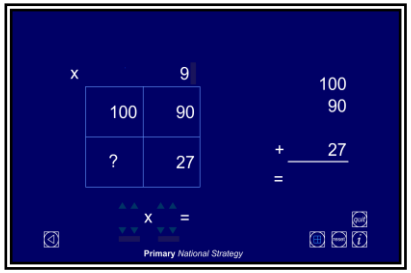
124 × 26 becomes

```
      1  2
      1  2  4
   ×     2  6
   -----------
      7  4  4
   2  4  8  0
   -----------
   3  2  2  4
   1  1
```

Answer: 3224

MULTIPLICATION GUIDELINES

Stage Four	Stage Five	Stage six



Use Multiplication grid ITP to assess understanding and application of the grid method by 'hiding' the question parts and 'revealing' some of the answer parts.

You can extend to using the grid method to multiply decimals.

Expanded Column Multiplication

Children should describe what they do by referring to the actual values of the digits in the columns. For example, the first step in 382 × 23 is 'three hundreds multiplied by twenty', not 'three times two', although the relationship 3 × 2 should be stressed.

Least significant first

382 x 23 =

300	80	2
X	20	3
		6 (3 x 2)
		240 (3 x 80)
		900 (3 x 300)
		40 (20 x 2)
		1600 (20 x 80)
		6000 (20 x 300)
		8786 300 + 80 + 2

382
x 23
6
240
900
40
1600
6000
8786

End of Year Objectives for Multiplication

Year 1 – practical problems that combine groups of 2, 5 or 10
Year 2 -represent multiplication as repeated + and arrays. Practical and informal written methods and vocabulary used to support multiplication alongside known facts and mental strategies. Understand and use '3 for free' for x and ÷ of the 2, 3,4,5,6, 8 and 10 times-tables. **Children should know multiplication tables 2, 5 and 10**
Year 3 – Describe the effect of Ux10, TUx10, Ux100, TU x 100. Practical and informal written methods for TU x U. **Children should know multiplication tables 2, 3, 4, 5, 8 and 10**
Year 4 – Derive and recall x and ÷ facts up to 12 x 12 and '3 for free' facts. Multiply numbers to 1000 by 10 and 100. Formal written layout and explain TU/HTU x U. **Children should know all multiplication tables to 12X12**
Year 5 – mentally multiply TU x U. Multiply whole numbers and decimals by 10, 100 and 1000. Formal written methods to multiply ThHTU x U, ThHTU x TU, U.t x U
Year 6 – mentally calculate TU x U, U.t X U. Formal written methods to multiply up to 4 digit by 2 digit and one digit with up to 2 decimal places.

As with addition and subtraction, before progressing through the stages of calculation:

Learning

▪ It is crucial to know or be able to derive key number facts:

- Understand and use doubling and halving
- ×/÷ 10 (as moving a place to the left/right NOT "add a zero" etc.!!)

▪ Place value and partitioning MUST be clearly understood and explained using the appropriate mathematical vocabulary.

Teaching

▪ The number line and the use of arrays must be modelled as images to support calculation from Reception to Year 6.
▪ Jottings must be modelled as a clear image/strategy for mental calculation.
▪ If the calculation should be possible mentally then do not give it to practise vertical calculation, e.g. 23 x 15 should not be calculated vertically. Consider use of numbers carefully.

Always present calculations horizontally in order to consider mental calculations first.

DIVISION GUIDELINES

Stage One

Prerequisite skills (based on the practical)
Understanding the language of half in different contexts.
Know halves of even numbers up to 10.

Sharing
Requires secure counting skills
-see counting and understanding number strand
Develops importance of one-to-one correspondence
See appendix for additional information on x and ÷ and aspects of number

Sharing – 6 sweets are shared between 2 people. How many do they have each?

● ● ● ● ● ●

Practical activities involving sharing, distributing cards when playing a game, putting objects onto plates, into cups, hoops etc.

Grouping
Sorting objects into 2s / 5s/ 10s etc.
How many pairs of socks are there?

There are 10 bulbs. Plant 5 in each pot. How many pots are there?
Jo has 10 Lego wheels. How many bicycles can she make?

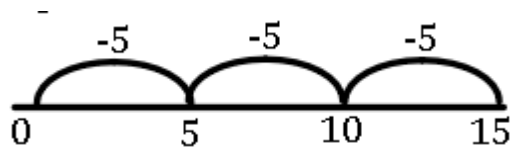
Stage Two

÷ = signs and missing numbers

6 ÷ 2 = ! ! = 6 ÷ 2
6 ÷ ! = 3 3 = 6 ÷ !
! ÷ 2 = 3 3 = ! ÷ 2
! ÷ ▽ = 3 3 = ! ÷ ▽

Grouping
Link to counting and understanding number strand
Count up to 100 objects by grouping them and counting in tens, fives or twos;…
Find one half, one quarter and three quarters of shapes and sets of objects
15 ÷ 5 can be modelled as:
There are 15 strawberries.
How many people can have 5 each? How many 5s make 15?

Repeated Subtraction
15 ÷ 5 can be modelled as repeated subtraction



In the context of money count forwards and backwards using 2p, 5p and 10p coins

Practical grouping e.g. in PE

12 children get into teams of 4 to play a game. How many teams are there?

Children should know that division is not commutative.

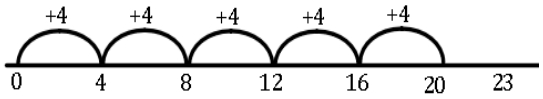
Stage Three

÷ = signs and missing numbers
Continue using a range of equations as in Stage 2 but with appropriate numbers.

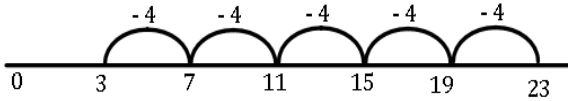
Understand division as sharing and grouping
24 ÷ 3 can be modelled as:
Sharing – 24 shared between 3

OR
Grouping - How many 3's make 24?

Remainders
23 ÷ 4 = 5r3
Grouping – How many 4's make 23, how many left over?
e.g.



Sharing - 23 shared between 4, how many left over?



DIVISION GUIDELINES

Stage Four

÷ = signs and missing numbers

Continue using a range of equations as in Stage 2 but with appropriate numbers.

Sharing, grouping, inverse of multiplication

60 ÷ 12 can be modelled as:

grouping – 12 subtracted repeatedly from 60 on a no. line, leading to subtracting 'groups' of 12.

sharing – sharing among 12, the number given to each person.

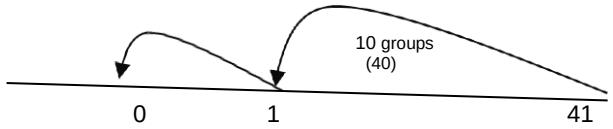
Use arrays to demonstrate the inverse relationship between multiplication and division - using known multiplication facts to work out division problems.

See http://nrich.maths.org/8773 for comprehensive article around using arrays for division

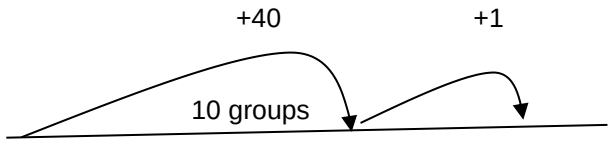
Chunking on a number line

Remainders can be shown effectively using a number line

41 ÷ 4 = 10 r1
+40
10 groups



41 ÷ 4 = 10 r1



41 = (10 x 4) + 1

Stage Five

Sharing and grouping

Continue to understand division as both sharing and grouping (repeated subtraction)

Pencil and paper procedures- Chunking

256 ÷ 7 lies between 210 ÷ 7 = 30 and
280 ÷ 7 = 40

256	
- 210	7 x 30
46	
- 42	7 x 6
4	

Answer: 36 remainder 4

Key Facts
1 x 7 =7
2 x 7 = 14
5 x 7 = 35
10 x 7 = 70

Quotients expressed as fractions or decimal fractions

61 ÷ 4 = 15 ¼ or 15.25

Short Division for one digit numbers

432 ÷ 5 becomes

```
   0  8  6  r2
   4  3
5 | 4  3  2
```

Answer: 86 remainder 2

Considering each column starting from the left.

Stage six

Sharing, grouping and remainders as Stage Five

Pencil and paper procedures- Chunking

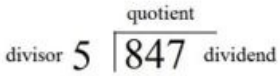
977 ÷ 36 is approximately 1000 ÷ 40 =

977	
- 720	36 x 20
257	
- 180	36 x 5
77	
- 72	36 x 2
5	

Answer: 27 5/36

Key Facts
1 x 36 = 36
2 x 36 = 72
5 x 36 = 180
10 x 36 = 360

Pencil and Paper procedures- Short Division Method



Short division with decimal and fractional remainders

496 ÷ 11 becomes

```
      4  5  r1
      5
1  1 | 4  9  6
```

Answer: $45\frac{1}{11}$

```
      1  7  8. 8
      3  4  4
5 | 8  9  4. 0
```

Pencil and Paper procedures- Long Division Method

This method should be reserved for the higher attaining children and only when finding a decimal remainders of two digit divisors.

End of Year Objectives for Division

Year 1 – practical problems that share into equal groups of 2, 5 or 10.
Year 2 – derive and recall division facts for 2, 5 or 10, represent division as repeated subtraction (grouping) and sharing. Practical and informal written methods and vocabulary used to support division, including remainders. To know that division is not commutative.
Year 3 – Practical and informal written methods for TU÷U. Understand and use '3 for free' for x and ÷ of the 2, 3, 4, 5, 6, 8 and 10 times-tables. Round remainders up or down, depending on the context.
Year 4 – Derive and recall x facts up to 12x12 and apply '3 for free' facts. Divide numbers to 1000 by 10 and 100. Develop and use formal
written layouts to record.
Year 5 – Divide whole numbers and decimals by 10, 100 and 1000. Divide numbers up to 4 digits by a one digit number using the formal written methods for division and interpret remainders appropriately for the context.
Year 6 – Divide numbers up to 4 digits by a 2 digit whole number using the formal written method of long division interpreting remainders as fractions, decimals, etc. Divide numbers up to 4 digits by a two digit number using the formal written methods for division and interpret remainders appropriately for the context.

As with multiplication, before progressing through the stages of calculation:

Learning

- It is crucial to know or be able to derive key number facts:
  - Understand and use doubling and halving
  - ×/÷ 10 (as moving a place to the left/right NOT "add a zero" etc.!!)
- Place value and partitioning MUST be clearly understood and explained using the appropriate mathematical vocabulary.

Teaching

- The number line and the use of arrays must be modelled as images to support calculation from Reception to Year 6.
- Jottings must be modelled as a clear image/strategy for mental calculation.
- If the calculation should be possible mentally then do not give it to practise vertical calculation, e.g. 24 ÷ 3 should not be calculated using short division. Consider use of numbers carefully.